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ABSTRACT

Outlined are the minimum requirements for a quinmester course of introduction to high school geometry. After a description of the course content and overall goals, further details are presented in nine sections. Each section gives performance objectives, textbook references, content (including lists of vocabulary and associated properties), and suggested teaching strategies. The material covered includes angles, parallels, perpendiculars, congruent and similar triangles, inequalities and constructions. There is an emphasis on the use of simple visual aids in developing the initial concepts. The pamphlet closes with sample posttest items and a bibliography of selected textbooks and audiovisual materials. (MM)

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AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

DADE COUNTY PUBLIC SCHOOLS

Mathematics: GEOMETRY 1 5218.21

SE 013 286

DIVISION OF INSTRUCTION • 1971

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

GEOMETRY 1

5218.21

(EXPERIMENTAL)

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for the

DIVISION OF INSTRUCTION
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Miami, Florida 33132
1971-72

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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested for a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the 1970-71 Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

The first of a two-quin sequence which introduces the student to all of the theorems usually included in high school geometry; emphasis is on the understanding and use of these theorems without proof. Develops concepts and skills relative to lines, angles, and triangles. Includes algebraic solution of some problems; sketching of 2-d and 3-d illustrations; interpreting data from illustrations; basic construction; and application of definitions, postulates, and theorems in problem solving.

Designed for the student who has mastered the skills and concepts of Algebra 1b.

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OVERALL GOALS

The student will

1. Acquire an appreciation of geometric forms in his everyday environment.
2. Use mathematical symbols, notations, and vocabulary peculiar to the study of geometry.
3. Improve his ability to reason informally.
4. Develop the ability to visualize spatial relationships and learn to sketch diagrams of these relationships.
5. Extend his understanding of measurement.
6. Develop reading techniques suitable for mathematics and science.
7. Extend his ability to do constructions with only a compass and straightedge.
8. Reinforce skills in computation.
9. Build a foundation for future development of formal proof.
10. Enrich his understanding of geometry through historical references.

KEY TO STATE ADOPTED REFERENCES

M - Moise, Edwin E. and Downs, Floyd L., Jr. Geometry. Reading, Massachusetts: Addison-Wesley Publishing Co., Inc., 1967.

L - Lewis, Harry. Geometry, A Contemporary Course. Princeton, New Jersey: D. Van Nostrand Company, Inc., 1968.

JD - Jurgensen, Donnelly, Dolciani. Modern Geometry. Boston: Houghton Mifflin Company, 1963.

A - Anderson, Garon, Gremillion. School Mathematics Geometry. Boston: Houghton Mifflin Company, 1966.

I. INTUITIVE INTRODUCTION TO TERMINOLOGY AND PROPERTIES

Performance Objectives

The student will

1. Separate the geometric properties of a given object from other properties which are physical or chemical.

2. Identify geometric terms which are commonly used in daily conversation.

3. Use geometric terms (oral and written) to describe physical objects in the world about him.

4. Read with understanding standard geometric notation for the sets: point, line, ray segment, length of a segment, plane angle, dihedral angle.

5. Sketch both two-dimensional and three-dimensional figures using the sets listed in objective 4.

6. Describe in standard geometric notation an object shown pictorially.

7. Identify and sketch the six geometric sets determined by two points.

8. Find the length of the segment determined by two given points.

State Adopted References

	M	L	JD	A
Ch.	1,2, 10	1	1	1,12

Other References

Dade County Experimental Project for Geometry, Level 1. S.A.G. 1.01, 1.02, 1.04

N.C.T.M. Geometry, Unit Four of Experiences in Mathematical Discovery. Washington, D.C.: National Council of Teachers of Mathematics, 1966.

I. INTUITIVE INTRODUCTION TO TERMINOLOGY AND PROPERTIES (continued)

Course Content

Give a brief introduction to geometric properties (size, shape, and position) and the use of geometric terminology in everyday life.

Develop intuitive understanding and use of standard geometric notation, as well as visualization of spatial relationships in two and three dimensions, where applicable to the following: point, line, plane, segment, length of a segment, ray, opposite rays, plane angle, dihedral angle, space.

Suggested Teaching Strategies

1. Have students maintain a notebook containing key ideas, concepts, and drawings.

2. Use inexpensive visual aids such as knitting needles, manilla folders, pencils, shoe box, index cards, straws, pipe cleaners, an orange, tiles, string, hangers, styrafoam, pick-up sticks, etc.

3. Have students make lists of objects found at home, outdoors, and in school which can be represented by point, line, plane, segment, etc.

4. Have students clip articles from newspapers and magazines and underline geometric terms.

5. Have students sketch objects found in the room, such as, the room itself, table, filing cabinet, desk, etc.

6. Use the NCTM booklet listed under references; it is very good for extra class activities.

II. FORMALIZED FUNDAMENTAL GEOMETRIC RELATIONSHIPS

Performance Objectives

The student will:

1. Use precise geometric language in describing relationships and properties.
2. State the conditions which determine a line, a plane, and space.
3. Identify sets (described verbally) which are coplanar and select all subsets which determine a plane.
4. Name the intersection of sets represented pictorially.
5. Identify sets (shown pictorially) which are: collinear, noncollinear, coplanar, noncoplanar, intersecting.
6. Determine whether or not a line lies in a given plane.
7. Identify the set used to separate: a line, a plane, space.
8. Determine whether or not a given set is convex.
9. Make a list of objects and classify each as a convex set or a set which is not convex.
10. Make sketches to illustrate all possibilities under each of the following conditions:

    a. 2 lines lie in a given plane
    b. 3 lines lie in a given plane
    c. A line and a plane intersect
    d. 2 half-planes have a common edge
    e. 2 planes intersect
    f. 3 planes intersect

State Adopted References

	M	L	JD	A
Ch.	2,3	1,7	1,3	2,3,4

Other References

Dade S.A.G. 1.05, 1.06, 1.10

II. FORMALIZED FUNDAMENTAL GEOMETRIC RELATIONSHIPS (continued)

Course Content

Formalize fundamental geometric concepts by the use of precise geometric language. Extend the visualization of spatial relationships in two and three dimensions, where applicable to:

Sets of points	Relationships	
point	collinear	noncollinear
line	coplanar	noncoplanar
plane	betweenness	separation
segment	convexity	nonconvexity
ray	intersecting lines	
opposite rays	non-intersecting lines (parallel and skew)	
half-line	non-intersecting planes	
half-plane	non-intersecting lines and planes	
half-space	intersections of other sets	
space	intersecting planes	

Associated Properties

The Distance Postulate.

Two points determine a line.

Every plane contains at least three noncollinear points.

Space contains at least four noncoplanar points.

If two points of a line lie in a plane, then the line lies in the plane.

Three collinear points lie in an infinite number of planes.

Three noncollinear points lie in exactly one plane.

A line, plane, and space are separated by a point, line, and plane, respectively.

Two lines separate a plane into three or four convex sets.

Three lines separate a plane into four, six, or seven convex sets.

Two planes separate space into three or four convex sets.

If two lines intersect, the intersection is exactly one point.

II. FORMALIZED FUNDAMENTAL GEOMETRIC RELATIONSHIPS

Course Content

Associated Properties (continued)

If a line intersects a plane not containing the line, the intersection is exactly one point.

If two planes intersect, the intersection is a line.

A plane is determined by: (1) three noncollinear points, (2) a point and a line not containing the point, (3) two intersecting lines, (4) two parallel lines.

Suggested Teaching Strategies

1. Make a first octant model from a cardboard box or from pegboard.

2. Use flexible wire and colored beads to discuss collinearity and betweenness, pointing out that these definitions apply only to points on a straight line.

3. Use drinking straws and pins to form a tripod, intersecting lines, etc.

III. ANGLES, PARALLELISM, AND PERPENDICULARITY IN A PLANE

Performance Objectives

The student will:

1. If given selected vocabulary items
   a. state the definition.
   b. make a sketch to illustrate.
   c. match items to suitable parts of a given sketch.

2. Use a protractor to
   a. construct an angle of any given measure.
   b. find the measure of any pictured angle.
   c. construct the supplement and/or the complement of a given angle.

3. Compute the measures of the remaining three of the four angles formed by two intersecting lines when given the measure of one angle.

4. Identify pairs of parallel lines in a figure having certain given angle measures.

5. If given a figure showing two parallel or two non-parallel lines cut by a transversal, name pairs of:
   a. congruent angles.
   b. supplementary angles.

6. Describe verbally and illustrate by sketching the relationship between two angles whose sides are:
   a. parallel.
   b. perpendicular (two cases are required).

7. Explain (orally or in writing) why two
   a. complementary angles are both acute.
   b. supplementary angles cannot both be acute.

8. If given a sketch showing two parallel lines cut by a transversal, identify and measure pairs of:
   a. congruent angles.
   b. supplementary angles.

State Adopted References

	M	L	JD	A
Ch.	4,6,9	2,4 6,8	1,4,5	5,7 10

Other References

Dade S.A.G. 1.10; Units II, IV, V

III. ANGLES, PARALLELISM, AND PERPENDICULARITY IN A PLANE (continued)

Course Content

Develop concepts related to the plane angle with minimal emphasis on proof.

Vocabulary

Plane angle
- sides
- vertex
- interior
- exterior
- acute
- obtuse
- right
- straight**
- reflex**
- linear pair

- supplementary
- complementary
- vertical
- congruent
- alternate exterior
- corresponding
- alternate interior

- interior on the same side of a transversal
- units of measure including degrees
- minutes
- seconds
- radians

Protractor

Perpendicular
- segments
- rays
- lines
- etc.

Parallel
- segments
- rays
- lines
- etc.

Transversal

** Varies from Moise point of view

Associated Properties

The size of an angle does not depend upon the length of its sides.

Every angle is congruent to itself. (Identity)

Two angles congruent to the same or congruent angles are congruent to each other. (Transitive property of congruence)

If two angles form a linear pair, they are supplementary.

If two angles are both congruent and supplementary, then each is a right angle.

Any two right angles are congruent.

Supplements of congruent angles are congruent.

Complements of congruent angles are congruent.

Vertical angles are congruent.

If two intersecting lines form one right angle, they form 4 right angles.

If 2 angles are complementary, then both are acute.

In a plane, if the sides of one angle are parallel to the sides of a second angle, the 2 angles are either congruent or supplementary.

III. ANGLES, PARALLELISM, AND PERPENDICULARITY IN A PLANE

Course Content

Associated Properties (continued)

Through a given external point there is only one parallel to a given line.

Conditions which determine parallel lines in a plane:

a. Two lines perpendicular to a third line are parallel to each other.
b. Two lines each parallel to a third line are parallel to each other.
c. If two lines are cut by a transversal and one pair of alternate interior (exterior) angles are congruent, then the lines are parallel.
d. If two lines are cut by a transversal and one pair of corresponding angles are congruent, then the lines are parallel.
e. If two lines are cut by a transversal and one pair of interior angles on the same side of the transversal are supplementary, then the lines are parallel.

If two parallel lines are cut by a transversal,

a. alternate interior angles are congruent;
b. corresponding angles are congruent;
c. interior angles on the same side of the transversal are supplementary.

Suggested Teaching Strategies

1. Refer to S.A.G., Unit II: Angles:

   a. "Parts of Angles" provides exercises on basic work with plane angles and dihedral angles.
   b. "Using the Protractor" is an introductory unit.
   c. "Using a Transit" uses a homemade transit.
   d. "Angle of Width" provides activities on measuring.
   e. "Vertical Angles" strengthens understanding of numerical relationships.
   f. "Numerical Exercises Using Definitions and Properties of Angles' is rich in supplementary problems.

2. Encourage students to discover by measurement the relationships in this unit.

3. Students enjoy making a transit; refer to Irving Allen Dodes, A Liberal Arts Approach to Mathematics.

IV. ANGLES, PERPENDICULARS, AND PARALLELS IN SPACE

Performance Objectives

The student will

1. Make sketches to illustrate each of the "Associated Properties."

2. State verbally the property illustrated when given a sketch.

3. Determine the projections into a plane of:

    a) a line
    b) a point
    c) a line segment
    d) an angle
    e) a pair of intersecting lines
    f) a pair of parallel lines
    g) a pair of skew lines

State Adopted References

	M	L	JD	A
Ch.	8,10	7,9	1,5,7	9,11

Other References

Dade S.A.G. 1.07; Unit II

Suggested Teaching Strategies

1. Refer to S.A.G., Unit II: Angles

    a. "Parts of Angles" provides exercises on basic work with dihedral angles.
    b. "Dihedral Angle Measure" contains many exercises.

2. Research on projective geometry and its relation to art, to map making, etc., makes a good extra credit project.

IV. ANGLES, PERPENDICULARS, AND PARALLELS IN SPACE (continued)

Course Content

Develop spatial concepts related to angles, parallelism, and perpendicularity.

Vocabulary

Dihedral angle (interior, exterior, vertical, right, measure, edge, face, plane angle)
Projection

Associated Properties

All plane angles of a given dihedral angle are congruent.

If a plane intersects two parallel planes, the intersection is two parallel lines.

If a line is perpendicular to one of two parallel planes, it is perpendicular to the other.

Two planes perpendicular to the same line are parallel.

Two lines perpendicular to the same plane are parallel.

Parallel planes are everywhere equidistant.

All plane angles of the same dihedral angle are congruent.

If a line is perpendicular to a plane, then every plane containing the line is perpendicular to the given plane.

If two planes are perpendicular, then any line in one of them, perpendicular to their line of intersection, is perpendicular to the other plane.

If a line and a plane are not perpendicular, then the projection of the line into the plane is a line.

V. CONSTRUCTION INVOLVING ANGLES AND SEGMENTS

Performance Objectives

Using only a compass and straight edge, the student will

1. Copy a given segment.
2. Find the sum and difference of measures of given segments.
3. Construct the perpendicular bisector of a given segment.
4. Pass a circle through 3 noncollinear points.
5. Erect a perpendicular to a given line at a point on the line.
6. Drop a perpendicular to a given line from an external point.
7. Copy a given angle.
8. Construct the sum and difference of measures of given angles.
9. Bisect a given angle.
10. Construct angles of special measures, such as: 90, 75, 60, 45, 30, 15, 150, etc.
11. Construct the parallel to a given line through a point external to the given line.
12. Make sketches to illustrate each of the "associated properties."
13. Divide a given segment into a given number of congruent segments.

State Adopted References

	M	L	JD	A
Ch.	4,5-5, 15	3,15	10	17

Other References

Dade Units II, IV, V

V. CONSTRUCTION INVOLVING ANGLES AND SEGMENTS (continued)

Course Content

Develop skills used in compass-straight edge constructions related to line segments and angles.

Vocabulary

Segment bisectors (midpoint, segment, ray, line, perpendicular)
Angle bisector

Associated Properties

Every segment has exactly one midpoint.

Every angle has exactly one bisector.

Through a given external point there is exactly one line parallel to the given line.

In a plane, through a given point on a given line there is exactly one line perpendicular to the given line.

In space, through a given point on a given line there are an infinite number of lines perpendicular to the given line.

Through an external point, there is exactly one line perpendicular to a given line.

In a plane, the perpendicular bisector of a given segment is the set of all points equidistant from the end points of the segment.

In space, the perpendicular bisecting plane of a segment is the set of all points equidistant from the end points of the segment.

The shortest segment joining a point to a line not containing the point is the perpendicular segment from the point to the line.

The shortest segment to a plane from an external point is the perpendicular segment from the point to the plane.

If a line is perpendicular to each of two intersecting lines at their point of intersection, then it is perpendicular to the plane determined by the intersecting lines.

If a line is perpendicular to a plane, then the plane contains every line perpendicular to the given line at its point of intersection with the given plane.

Two lines perpendicular to the same plane are parallel.

In a plane, two lines perpendicular to the same line are parallel to each other.

In any triangle, the sum of the measures of the angles is 180.

VI. CONGRUENT TRIANGLES

Performance Objectives

The student will

1. Define and illustrate items selected from the vocabulary list.

2. Label corresponding parts of given marked figures.

3. Name congruent triangles using notation which shows corresponding vertices of a given marked figure.

4. State the postulate which justifies the congruence of two marked figures.

5. Identify all corresponding parts of given congruent triangles, where the correspondence is shown:

   a. pictorially
   b. by letters without a figure

6. State conclusions which can be derived from the given facts:

   a. Two sides of a triangle are congruent.
   b. Two angles of a triangle are congruent.
   c. Three sides of a triangle are congruent.

7. With only a straight edge and compass copy a given triangle using each of the following methods:

   a. SSS
   b. SAS
   c. ASA

*8. Construct a triangle with a straight edge and compass when given:

   a. SSS
   b. ASA
   c. SAS
   d. SAA
   e. HL
   f. HA

* Optional

State Adopted References

	M	L	JD	A
Ch.	5,15	5,15	2,6, 10	17

Other References

Dade Unit VI

VI. CONGRUENT TRIANGLES (continued

Course Content

Develop precise vocabulary and geometric notation related to triangles and congruence; postulate and/or develop inductively.

Vocabulary

Polygon
Triangle
- side
- angle
- vertex
- adjacent sides
- included angles
- included sites
- interior and exterior of triangle (intuitive)

Kinds of triangles and related parts:
- scalene
- isosceles
- equilateral
- equiangular
- right
- obtuse
- acute
- legs
- hypotenuse
- vertex angle
- base
- base angles

One-to-one correspondence
- corresponding sides
- corresponding angles
- congruent sides
- congruent angles
- congruent triangles

Associated Properties

Two polygens are congruent if, for some pairing of their vertices, each side and each angle of one polygon is congruent to the corresponding part of the other polygon.

A triangle is a three-sided polygon.

The correspondence ABC $\leftrightarrow$ ABC is called the identity congruence.

If AB = AC, then the correspondence ABC $\leftrightarrow$ ACB is called a self congruence, which is not an identity congruence.

Congruence postulates and theorems: SSS, SAS, ASA, SAA, HL, HA

If two sides of a triangle are congruent, the angles opposite those sides are congruent; and, conversely, if two angles of a triangle are congruent, the sides opposite those angles are congruent.

Every equilateral triangle is equiangular and vice versa.

Constructions

Copy a triangle if given: SSS, ASA, SAS, SAA, HL, HA

VII. SIMILAR TRIANGLES

Performance Objectives

The student will

1. Find the missing term of a proportion when given the other three terms.
2. Find the geometric and arithmetic mean of two terms given in numerical or literal form.
3. Determine whether figures in a given marked diagram are similar; and if so, identify the property which justifies the conclusion.
4. List all proportions determined by a line intersecting two sides of a triangle and parallel to the third side.
5. Determine whether a segment is parallel to one side of a triangle if given certain proportional parts.
6. Identify the proportions determined by a bisector of an angle of a triangle.
7. Solve for the missing measures of a right triangle if given a drawing showing the altitude to the hypotenuse.
8. If given the ratio of corresponding parts of similar figures, apply the proper formulas to find:
   a. perimeter
   b. area
   c. volume
9. Apply the Pythagorean Theorem to find the missing measures of the sides of a given right triangle.
10. Use the converse of the Pythagorean Theorem to determine whether a triangle is a right triangle.
11. Apply the appropriate formula to solve for the missing parts in special right triangles:
    a. 30-60-90
    b. 45-45-90

State Adopted References

	M	L	JD	A
Ch.	11,12	11	7,10	13,14

Other References

Dade Unit VI

VII. SIMILAR TRIANGLES (continued)

Course Content

Develop precise vocabulary and geometric notation related to similarity, with emphasis on triangles. Postulate and/or develop inductively properties of similar triangles and related properties of special triangles.

Vocabulary

Ratio
- proportion
- means
- extremes

geometric mean
arithmetic mean

Similar polygons
- similar triangles
- proportional segments

Projection

Associated Properties

Two polygons are similar if, for some pairing of their vertices, corresponding angles are congruent and corresponding sides are in proportion.

Similarity theorems: AAA, AA, SAS, SSS

If a line is parallel to one side of a triangle and intersects the other two sides, then a) it divides the sides into proportional segments; b) it cuts off segments proportional to these sides and conversely.

If a ray bisects one angle of a triangle, it divides the opposite side into segments which are proportional to the adjacent sides.

Transitivity of similarity with respect to triangles.

The altitude to the hypotenuse of a right triangle forms two triangles which are similar to the given triangle and similar to each other.

The altitude to the hypotenuse of a right triangle is the mean proportional (geometric mean) between the segments into which it divides the hypotenuse.

A leg of a right triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse.

If two triangles are similar, then the ratio of their areas is equal to the square of the ratio of any two corresponding sides, altitudes, or medians.

VII. SIMILAR TRIANGLES

Course Content

Associated Properties (continued)

If two triangles are similar, then the ratio of their perimeters is equal to the ratio of any two corresponding sides, altitudes, or medians.

Pythagorean Theorem and its converse.

Theorems on special triangles: 30-60-90 and 45-45-90.

VIII. INEQUALITIES

Performance Objectives

The student will

1. Use the order properties (transitivity, addition, multiplication by a positive number) in the solution of simple inequalities.
2. Find the missing angle measures of a triangle if given measures of an exterior angle and one remote interior angle.
3. Use exterior angle properties of a triangle and transitivity to establish relative sizes of angles in a given figure.
4. Determine the largest angle of a triangle if given measures of the sides.
5. Determine the longest side of a triangle if given the measures of the angles.
6. Determine the largest angle of a figure partitioned into triangles if given appropriate information.
7. Determine the longest side of a figure partitioned into triangles if given appropriate information.
8. Solve numerical problems involving the definition of distance from a point to a line or plane.
9. Determine whether the given measures could be the lengths of the sides of a triangle.
10. Use the "Hinge Theorem" and its converse to compare lengths of segments and measures of angles.

State Adopted References

	M	L	JD	A
Ch.	7	16	pp. 17, 90,91, 97,101	8

VIII. INEQUALITIES (continued)

Course Content

Develop precise vocabulary and geometric notation related to triangles and inequalities. Postulate and/or develop inductively associated properties.

Vocabulary

Order of inequality
exterior angle of a triangle
remote interior angle of a triangle

Associated Properties

Properties of order:

trichotomy	addition
transitivity	multiplication (by a positive number)

An exterior angle of a triangle is larger than each of its remote interior angles.

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

If a triangle has one right angle, then its other angles are acute.

If two sides of a triangle are not congruent, then the angles opposite those sides are not congruent and the larger angle is opposite the longer side; and, conversely.

The shortest segment joining a point and a line is the perpendicular segment.

The distance from a point to a line or from a point to a plane is the length of the perpendicular segment.

Triangle Inequality: The sum of the lengths of any two sides of a triangle is greater than the third side.

"Hinge Theorem": If two sides of one triangle are congruent to two sides of a second triangle and the included angle of the first triangle is larger than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle; and, conversely.

IX. CONSTRUCTIONS INVOLVING TRIANGLES

Performance Objectives

Using only a compass and straightedge, the student will construct

1. Isosceles triangles
2. Equilateral triangles
3. Right triangles
4. Angle bisectors of triangles
5. Perpendicular bisectors of the sides of a triangle
6. Medians of a triangle
7. Bimedians of a triangle
8. Altitudes of a triangle
9. Inscribed circle of a triangle
10. Circumscribed circle of a triangle

State Adopted References

	M	L	JD	A
Ch.	15	11,15	10	13,14, 17

Course Content

Develop precise vocabulary and geometric notation related to constructions involving triangles and special set of points related to triangles.

Vocabulary

Sets of points related to a triangle:

angle bisector
perpendicular bisector of a side
median
bimedian
altitude
circumcenter
incenter
orthocenter
centroid
circumscribed circle
inscribed circle

Associated Properties

The medians of every triangle are concurrent at a point two-thirds of the distance from one vertex to the mid-point of the opposite side.

The segment joining the midpoints of two sides of a triangle is parallel to the third side and one-half as long.

SAMPLE POSTTEST ITEMS

For a final test, choose an appropriate number of test questions from the following collection.

I. From the list at the right, select the one best choice which matches each identifying sentence or phrase in items 1 - 10. Write the letter of your choice in the appropriate blank. A letter may be used more than once.

1. An undefined term
2. Every angle has two
3. Does not contain its edge
4. Means "at least one"
5. The union of three rays
6. A property of betweenness
7. Set common to two sets
8. Always equilateral
9. Angles having equal measures
10. Angle whose measure is less than 90 degrees

a. Acute angle
b. Between
c. Bisector
d. Collinearity
e. Congruent
f. Converse
g. Coordinate
h. Corollary
i. Diagonal
j. Dihedral angle
k. Exactly one
l. Existence
m. Exterior angle
n. Half plane
o. Hypothesis
p. Intersection
q. Isosceles
r. Plane
s. Linear pair
t. Ray
u. Rhombus
v. Transversal
w. Triangle
x. Union

II. Write TRUE if the statement is true. If the statement is false, write the replacement for the underlined word(s) or symbol(s) which will make the statement true.

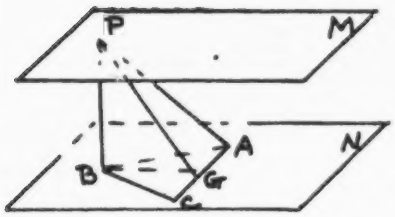
11. The longest side of a right triangle is called the hypothesis.
12. Two lines in space are either parallel, intersecting, or skew.
13. If two planes are each parallel to the same line, the two planes are perpendicular to each other.
14. In $\triangle ABC$, if $m\measuredangle A > m\measuredangle C$, then $AC > BC$.
15. If two lines are each perlendicular to a third line, the two lines are perpendicular.
16. In a plane, all points equidistant from two given points lie on the median of the segment joining the two points.
17. Two planes may separate space into four half-planes.
18. Given a correspondence $ABC \leftrightarrow BCA$ of the vertices of a triangle, if $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{CA}$, then $\triangle ABC$ is scalene.
19. In a plane, if a line is perpendicular to one of two parallel lines, it is parallel to the other.

III. Find the solution to each problem; state the solution in simple radical form.

20. The measure of an angle is six times the measure of its supplement. What is the measure of the angle?
21. What is the measure of the complement of an angle of $82 + x$ degrees.
22. The measure of an exterior angle of a triangle is 106° and the measure of one angle of the triangle is 68°. What are the measures of the other two angles of the triangle?
23. In $\triangle ABC$, $\measuredangle C$ is a right angle. $\overline{CM}$ is the median to the hypotenuse, and $\overline{CD}$ is the altitude to the hypotenuse. If $m\measuredangle A = 18$, what is $m\measuredangle MCD$?

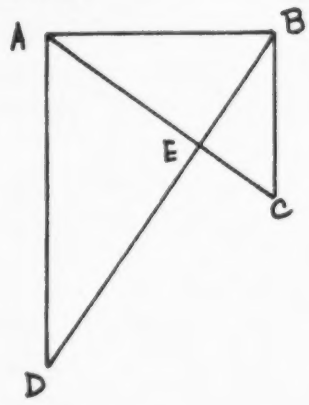
Sample Posttest Items - Continued

III. Continued



24. In the figure, plane M || plane N, $\overleftrightarrow{PB}$ N, and PG is the perpendicular bisector of $\overline{CA}$. If m∠C-PB-A = 60°, what is m∠BAC?

25. Two triangles are similar and their areas are 81 and 36. What is the ratio of a pair of corresponding sides?



26 - 28. In the given plane figure, $\overline{AD} \perp \overline{AB}$, $\overline{BC} \perp \overline{AB}$, AB = 6, BE = 4, and DE = 6.

26. What is the numerical value of $\frac{BC}{AD}$?

27. What is the numerical value of $\frac{a\triangle BCE}{a\triangle DAE}$?

28. What is the numerical value of $\frac{a\triangle ABC}{a\triangle ABD}$?

29 - 31. In △ABC, ∠A is a right angle and $\overline{AD} \perp \overline{BC}$ with D on $\overline{BC}$. BD = 16 and CD = 20.

29. What is AD?
30. What is AB?
31. What is AC?

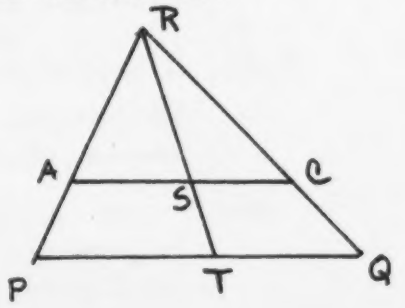
32. Solve for x: $\frac{x+3}{4x} = \frac{14}{40}$

33. Solve for x: $\frac{15ma}{4xy} = \frac{18ab}{4my}$

34. Complete: If $\frac{36}{5}.5 = 4x$, then $\frac{x}{5} =$ _____.

35. Two similar triangles have a pair of corresponding sides of lengths 2.5 and 8.6 respectively. What is the ratio of their perimeters?

36. Two similar right triangles have hypotenuses of lengths 5 and 10. What is the ratio of the areas of the two triangles.



37 - 38. In the given figure, $\overline{AC} \parallel \overline{PQ}$.

37. If AR = 6, AP = 2, and CQ = 5, then what is RC?

38. If PR = 33, AP = 10, and ST = 8, then what is RS?

Sample Posttest Items - Continued

III. Continued

39. Two angles of one triangle have measures 30 and 100, and two angles of another triangle have measures 30 and 50. Are the triangles similar?

40. In $\triangle PQR$, $\measuredangle Q$ is a right angle and the medians intersect at S. If $PR = 12$ and $PQ = 9$, how long is $\overline{QS}$?

IV. Complete each sentence with the correct symbol(s) or word(s).

41. A line and a plane are perpendicular if they intersect and if every line lying in the plane and passing through the point of intersection is _____.

42-43. If two angles of a triangle are not congruent, then the sides opposite them are _____ and the _____ is opposite the larger side.

44-45. Every line separates the points of a plane not on the line into two _____ sets, and if P is a point in one set and Q is a point in theother set, then the segment PQ _____.

46-47. The segment between the midpoints of two sides of a triangle is _____ to the third side and _____.

48. If two half-planes have the same edge but do not lie in the same plane, then the union of the half-planes and their common edge is called _____.

49. Two or more lines are said to be _____ if they share a common point.

50. The point of intersection of the angle bisectors of a triangle is called the _____.

51. The point of intersection of the _____ of a triangle is called the centroid.

52-53. The intersection of the _____ of a triangle and the intersection of the _____ of a triangle may each lie in the exterior of the triangle.

54-55. A correspondence between two triangles is a similarity if the _____ sides are proportional and the corresponding _____ are congruent.

56-57. An example of a correspondence in which corresponding angles are congruent but the correspondence is not a similarity is that between a _____ and _____.

58. If m is the arithmetic mean between the two positive numbers r and s, then m _____.

V. Select the letter(s) corresponding the choice(s) which correctly answer the question. Give <u>all</u> correct choices.

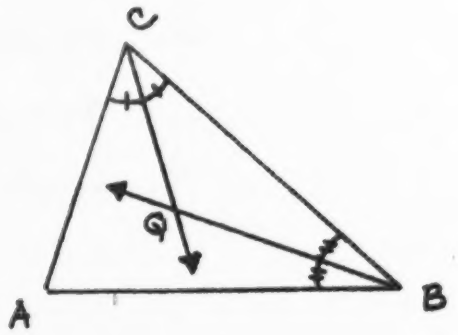
59. Which of the following are not convex sets?
(a) a square (b) a ray (c) a line (d) a triangle (e) a plane

60. Which congruence conditions, abbreviated in the choices below, are sufficient for proving the triangles congruent?
(a) SAS (b) AAS (c) SSS (d) SSA (e) AAA (f) ASA

61. The projection of an angle into a plane may not be which of the following sets?
(a) a line (b) a ray (c) a point (d) a segment (e) an angle

62. Given two planes in space. Into how many regions can they separate space?
(a) three (b) four (c) six (d) seven (e) eight

63. The figure illustrates the first steps in constructing a certain circle associated with $\triangle ABC$. What should be the next step in the construction?
(a) Let $\overleftrightarrow{CQ}$ intersect $\overline{AB}$.
(b) Construct the bisector of $\angle CQB$.
(c) Construct a perpendicular from Q to $\overline{AC}$.
(d) Draw the circle with center Q and radius $\overline{QB}$.
(e) Draw a circle tangent to $\overline{AC}$.



64. Two altitudes of a triangle intersect in the exterior of the triangle. The triangle must be
(a) right (b) isosceles (c) equilateral (d) obtuse (e) acute

VI. Print TRUE if the statement is true; print FALSE if the statement is false.

65. The ancient Greeks used a protractor to do geometry.

66. The set of all points in the interior of an angle which are equidistant from the sides of the angle is defined as the bisector of the angle.

67. Using only a compass and straightedge, it is possible to construct a $5\frac{1}{2}^{\circ}$ angle.

68. In space, the set of all points equidistant from the end points of a segment is the midpoint of the segment.

Sample Posttest Items - Continued

VI. Continued

69. The set of all points in a plane which are vertices of equilateral triangles having a given segment as base is a line.

70. Using only a compass and straightedge, it is not possible to construct a 105° angle.

VII. Use only a compass and straightedge to construct:

71. A scalene triangle ABC.

72. The median to side $\overline{BC}$ in ΔABC.

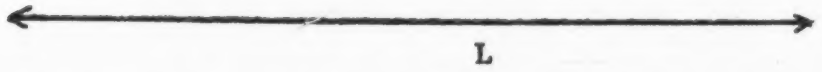
73. The bisector of $\angle C$ in ΔABC.

74. The perpendicular bisector of side $\overline{AC}$ in ΔABC.

75. The altitude to side $\overline{AB}$ in ΔABC.

76. A line through point P which will be parallel to line L.

. P



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1. Wilcox, Marie S. Geometry a Modern Approach. Reading, Mass: Addison-Wesley Publishing Co., 1968.
   Good for quiz and test materials as well as general spacial concepts.

2. Dodes, Irving A. Geometry. New York: Harcourt, Brace and World, Inc., 1965.
   Good for practical and commercial applications of geometry.

3. Smith, Rolland R. and Ulrich, James F. Geometry, A Modern Course. New York: Harcourt, Brace and World, Inc., 1964.
   Excellent chapter on constructions and three-dimensional geometry. (Chapter 3)

4. Keedy, Mervin, L. and Jameson, Richard, E. Exploring Geometry. New York: Holt, Rinehart, and Winston, Inc., 1967.
   Excellent diagrams and explanations of three-dimensional figures. Teachers commentary gives helpful additional explanations. Very good source book for entire quin.

5. Ulrich, James, F. and Payne, Joseph N. Geometry. New York: Harcourt, Brace and World, Inc., 1969.
   Chapters one and two excellent on definitions and fundamental relationships. Chapter five helpful with congruent triangles.

6. Goodwin, Wilson A. and Vannatta, Glen, D. Geometry. Columbus, Ohio: Charles E. Merrill Publishing Co., 1970.
   Chapters two, three, five, seven, eight, and ten very helpful in this quin for supplementary topics and exercises.

7. Fehr, Howard F. and Carnahan, Walter H. Geometry. Boston, Mass: D.C.Heath and Co., 1961.
   Chapters one, three, five, eight, nine, fifteen excellent for test questions and quis material.

8. Munro, Thomas and Wilson, Catherine M. Tenth Year Mathematics. New York: Oxford Book Co., 1960.
   Good basic review material. Excellent for additional questions for homework or tests.

AUDIO-VISUAL MATERIAL
Films - By request only through the school library

Cat. Number

1-01493	Geometry and You (10 min.)
1-01506	Similar Triangles (12 min.)
1-01348	Triangles: Types and Uses (11 min.)

Annotated Bibliography, Audio-Visual Material (continued)

Curriculum Full-Color Filmstrips by Educational Projections, Inc.

362 Introduction to Plane Geometry. Defines angle, point, line, plane. Uses of geometry.

363 Lines and Angles - I. Emphasizes kinds of angles.

364 Lines and Angles - II. Emphasizes perpendicular lines and distance.

365 Lines and Their Relationships. Emphasis on parallel lines, alternate interior angles, corresponding angles, and transversals.

366 Triangles. Types of triangles, median, angle bisector, altitude, hypotenuse, exterior angle.

370 Intersection of Straight Lines. Distance between two points. Midpoint. Line perpendicular from a point to a given line.

376 Geometrical Logic. To follow congruent triangles. Induction.

372 Locus. Very good explanation and examples.

Filmstrips by S. V. E.

A-541-2 Introduction to Plane Geometry. a. Good coverage of angles. b. Adjacent and vertical angles. c. Practical application of angle measurement.

Filmstrips by McGraw-Hill Book Co.

9 Indirect measurement. (Excellent) Use after study of right triangles.

OVERHEAD VISUALS
Overhead Visuals for Geometry by Jurgensen, Donnelly, Dolciani

Vol. I Elements of Geometry

Vol. II Angle Relationships, Perpendicular Lines

Vol. III Parallel Lines and Planes
Congruent Triangles

Vol. V Construction and Loci